Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

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P 6 6 8 0 0 A 0 1 2 8

Pearson

1. A van of mass 900 kg is moving along a straight horizontal road.

At the instant when the speed of the van is $v\,\text{m s}^{-1}$, the resistance to the motion of the van is modelled as a force of magnitude $(500 + 7v)$ N.

When the engine of the van is working at a constant rate of 18 kW, the van is moving along the road at a constant speed $V\,\text{m s}^{-1}$

(a) Find the value of V.

(5)

Later on, the van is moving up a straight road that is inclined to the horizontal at an angle θ, where $\sin\theta = \dfrac{1}{21}$

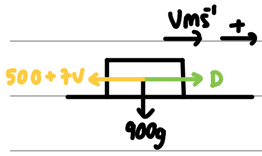
At the instant when the speed of the van is $v\,\text{m s}^{-1}$, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude $(500 + 7v)$ N.

The engine of the van is again working at a constant rate of 18 kW.

(b) Find the acceleration of the van at the instant when $v = 15$

(4)

(a) Diagram



Since the speed is constant we can use $\Sigma F_x = 0$

$D = 500 + 7v$ M1

To get D we will use Power.

Formula for Power:

Power (W) — $P = Dv$ — Driving force (N), velocity (ms⁻¹)

$P = 18\text{ kW} \xrightarrow{\times 1000} 18\,000\text{ W}$
$D = D$
$v = v$

Substitute: $18000 = Dv$

$D = \dfrac{18\,000}{v}$ M1

Substitute our D and solve for v:

$\dfrac{18\,000}{v} = 500 + 7v$ A1

$0 = 7v^2 + 500v - 18\,000$ M1

Use Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

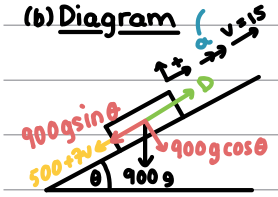
$v = \dfrac{-500 \pm \sqrt{500^2 - 4(7)(-18\,000)}}{14}$

$v = 26.309 \longrightarrow \boxed{V = 26.3\text{ ms}^{-1} \text{ to } 3\text{sf}}$ A1

Question 1 continued

(b) Diagram

we want this!



Since it's accelerating, use $\Sigma F_x = ma$

$D - (500 + 7v) - 900g\sin\theta = 900a$ M1

To get D we will use Power.

Formula for Power:

Power (W) — $P = Dv$

Driving force (N), velocity (ms^{-1})

P: 18kW $\xrightarrow{\times 1000}$ 18000W

$D = D$

$v = 15ms^{-1}$

Substitute: $18000 = 15D$

$D = 1200$ N A1

Substitute our D and solve for a:

$1200 - 500 - 7(15) - \frac{900g}{21} = 900a$ A1

$700 - 105 - 420 = 900a$

$\frac{175}{900} = a$

$\boxed{a = 0.194ms^{-2} \text{ to 3sf}}$ A1

Question 1 continued

P 6 6 8 0 0 A 0 4 2 8

Question 1 continued

(Total for Question 1 is 9 marks)

P 6 6 8 0 0 A 0 5 2 8

2. Two particles, A and B, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Particle A has mass $5m$ and particle B has mass $3m$.

The coefficient of restitution between A and B is e, where $e > 0$

Immediately **after** the collision the speed of A is v and the speed of B is $2v$.

Given that A and B are moving in the same direction after the collision,

(a) find the set of possible values of e.

(8)

Given also that the kinetic energy of A immediately after the collision is 16% of the kinetic energy of A immediately before the collision,

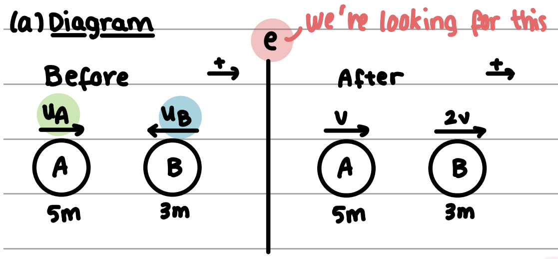
(b) find

(i) the value of e,

(ii) the magnitude of the impulse received by A in the collision, giving your answer in terms of m and v.

(6)

(a) Diagram



We can use the conservation of linear momentum to get this.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A \underbrace{u_A}_{\text{initial velocity}} + m_B u_B = m_A \underbrace{v_A}_{\text{final velocity}} + m_B v_B$

Substitute: $5\cancel{m}(u_A) + 3\cancel{m}(-u_B) = 5\cancel{m}(v) + 3\cancel{m}(2v)$ cancel m's M1A1

$5u_A - 3u_B = 5v + 6v$

$5u_A - 3u_B = 11v$ Eq. 1

We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $\underbrace{e}_{\text{coefficient of restitution}}\underbrace{(u_A - u_B)}_{\text{initial speed}} = \underbrace{v_B - v_A}_{\text{final speed}}$

Substitute: $e(u_A - -u_B) = 2v - v$ M1 A1

$e(u_A + u_B) = v$ Eq. 2

Question 2 continued

Solve simultaneously Eq1 and Eq2:

For u_A:

$$5u_A - 3u_B = 11v$$

$$eu_A + eu_B = v \quad |\times \tfrac{3}{e}| \qquad 3u_A + 3u_B = \frac{3v}{e} \quad +$$

use elimination method

$$8u_A = 11v + \frac{3v}{e}$$

factor out $\frac{v}{e}$

$$8u_A = \frac{v}{e}(11e+3)$$

$$u_A = \frac{v}{8e}(11e+3) \quad \text{M1}$$

For u_B:

$$5u_A - 3u_B = 11v$$

$$eu_A + eu_B = v \quad |\times -\tfrac{5}{e}| \qquad -5u_A - 5u_B = -\frac{5v}{e} \quad +$$

use elimination method

$$-8u_B = 11v - \frac{5v}{e}$$

factor out $-\frac{v}{e}$

$$-8u_B = -\frac{v}{e}(5-11e)$$

$$u_B = \frac{v}{8e}(5-11e) \quad \text{A1}$$

Since $e > 0$, $u_A > 0$ for all e (moves in $\xrightarrow{+}$ direction).

We want u_B to move in the negative direction, but we already assumed this in our diagram

$\therefore$ we want $u_B > 0$.

$$u_B > 0 \quad \text{M1}$$

$$\frac{v}{8e}(5-11e) > 0$$

$$5 - 11e > 0$$

$$\frac{5}{11} > e$$

Since e is a coefficient of restitution, $0 \le e \le 1$

So the full range for e:

$$0 < e < \frac{5}{11} \quad \text{A1}$$

Question 2 continued

(b) We are given that $KE_{Fin.} = 16\%$ of $KE_{Init.}$

$\therefore KE_F = 0.16KE_I$

Formula for Kinetic Energy:

$$KE = \frac{1}{2}\underbrace{m}_{\text{mass}}\underbrace{v}_{\text{velocity}}^2$$

Substitute:

$$\frac{1}{2}mv^2 = \frac{16}{100} \times \frac{1}{2}mu^2$$

M1 A1 $\cancel{\frac{1}{2}}\cancel{(5m)}(v)^2 = \frac{16}{100} \times \cancel{\frac{1}{2}}\cancel{(5m)}\left(\frac{v}{8e}(11e+3)\right)^2$ cancel $\frac{1}{2} \times 5m$

i. Solve for e:

$$\cancel{v^2} = \frac{16}{100}\left(\frac{\cancel{v^2}}{64e^2}(11e+3)^2\right) \quad \text{cancel } v^2$$

$$\frac{100}{16} = \frac{1}{64e^2}(11e+3)^2$$

$$\overset{4}{\cancel{64}}e^2 \times \frac{100}{\cancel{16}} = (11e+3)^2$$

$$400e^2 = (11e+3)^2$$

$$\sqrt{400e^2} = \sqrt{(11e+3)^2} \quad \text{square root both sides}$$

$$20e = 11e + 3$$

A1 $9e = 3 \rightarrow e = \frac{1}{3}$ value of e

ii. Impulse is the change in momentum

Formula for change in momentum:

$$I = \Delta\text{momentum} = \overset{\text{mass}}{m}v_{\text{final}} - mv_{\text{initial}}$$

velocity

$\therefore$ Formula for Impulse: $I = m(v-u)$

Substitute: M1 $I = -5m(v - u_A)$ Impulse on A acts in the negative direction, $\therefore$ we use a negative sign

A1 $= -5m\left(v - \frac{v}{8e}(11e+3)\right)$

$$= -5mv\left(1 - \frac{1}{8\times\frac{1}{3}}\left(11\times\frac{1}{3}+3\right)\right)$$

$$= -5mv\left(1 - \frac{3}{8}\left(\frac{11}{3}+3\right)\right)$$

$$= -5mv\left(1 - \frac{\cancel{3}}{\cancel{8}_2}\times\frac{\cancel{20}^5}{\cancel{3}}\right)$$

$$= -5mv \times -\frac{3}{2}$$

A1 $I = \frac{15}{2}mv$ magnitude of Impulse

Question 2 continued

(Total for Question 2 is 14 marks)

3. [*In this question*, **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

A smooth uniform sphere P has mass 0.3 kg. Another smooth uniform sphere Q, with the same radius as P, has mass 0.5 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of P is $(u\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$, where u is a positive constant, and the velocity of Q is $(-4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$

At the instant when the spheres collide, the line joining their centres is parallel to **i**.

The coefficient of restitution between P and Q is $\frac{3}{5}$ $= e$

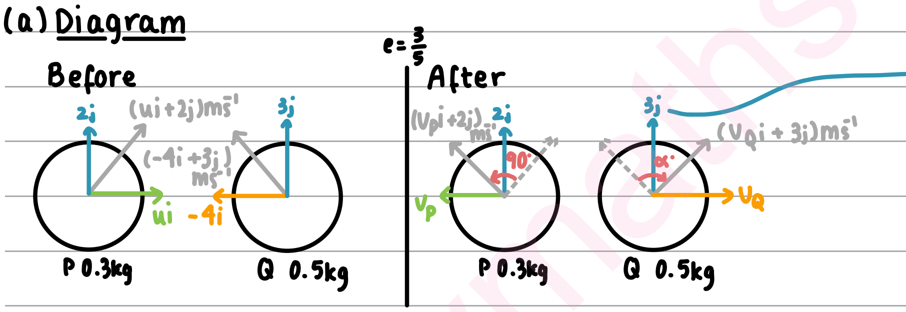
As a result of the collision, the direction of motion of P is deflected through an angle of 90° and the direction of motion of Q is deflected through an angle of $\alpha°$

(a) Find the value of u **(8)**

(b) Find the value of α **(5)**

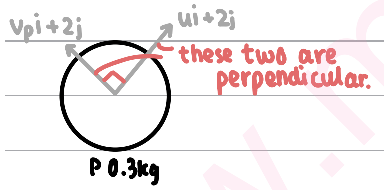
(c) State how you have used the fact that P and Q have equal radii. **(1)**

(a) Diagram



Perpendicularly to the line of centers the velocity stays the same B1

Since we know P is deflected by 90°, we can look at P geometrically/algebraically to get V_p.



let's get their "gradients" and use $m_1 \times m_2 = -1$

$\frac{2}{u} \times \frac{2}{V_p} = -1$ $\frac{\text{Rise}}{\text{Run}} = \frac{j}{i}$

$\frac{4}{uV_p} = -1$

$V_p = -\frac{4}{u}$

$\therefore$ velocity of P after is $(-\frac{4}{u}\mathbf{i}+2\mathbf{j})\,\text{ms}^{-1}$ B1

Question 3 continued

Parallel to the line of centers:

We can use the conservation of linear momentum to get an equation.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

u_A initial velocity, v_A final velocity

Substitute: $0.3u + 0.5(-4) = 0.3\left(-\frac{4}{u}\right) + 0.5v_Q$

$0.3(u) + 0.3\left(\frac{4}{u}\right) = 0.5v_Q + 0.5(4)$

$0.3\left(u + \frac{4}{u}\right) = 0.5(v_Q + 4)$ Eq.1 M1A1

We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution, initial speed, final speed

Substitute: $\frac{3}{5}(u - -4) = v_Q - -\frac{4}{u}$

$\frac{3}{5}(u + 4) = v_Q + \frac{4}{u}$ Eq.2 M1A1

Solve simultaneously Eq1 and Eq2:

M1 $0.3\left(u + \frac{4}{u}\right) = 0.5(v_Q + 4)$ we want to eliminate this value since we are looking for u

$0.3u + \frac{1.2}{u} = 0.5v_Q + 2$

$\frac{3}{5}(u+4) = v_Q + \frac{4}{u}$ $|\times -0.5|$ $-\frac{3}{10}(u + 4) = -0.5v_Q - \frac{2}{u}$ +

$\cancel{0.3u} + \cancel{\frac{1.2}{u}} - \cancel{0.3u} - \cancel{\frac{1.2}{u}} = 2 - \frac{2}{u}$

$0 = 2 - \frac{2}{u}$

$\frac{2}{u} = 2 \rightarrow u = 1$ A1

(b) Velocity of Q after is $(v_Q\mathbf{i} + 3\mathbf{j})\,\text{ms}^{-1}$.

Get v_Q: $\frac{3}{5}(u+4) - \frac{4}{u} = v_Q$ Substitute u=1 from (a)

$\frac{3}{5}(5) - 4 = v_Q$

$v_Q = -1$ B1

$\therefore (-\mathbf{i} + 3\mathbf{j})\,\text{ms}^{-1}$ B1

Use scalar product formula to get angle α. M1

$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}| \times |\underline{b}|}$

$\alpha = \cos^{-1}\left(\frac{\begin{pmatrix}-4\\3\end{pmatrix} \cdot \begin{pmatrix}-1\\3\end{pmatrix}}{5 \times \sqrt{1^2+3^2}}\right)$

$\alpha = \cos^{-1}\left(\frac{4+9}{5\sqrt{10}}\right)$ A1

$\alpha = 34.7°$ to 3sf angle α A1

Question 3 continued

(c) the line of centers is parallel to the surface the spheres are moving on, so the impulse acts parallel to the surface B1

Question 3 continued

(Total for Question 3 is 14 marks)

4. A particle P has mass 0.5 kg. It is moving in the xy plane with velocity $8\mathbf{i}\,\mathrm{m\,s^{-1}}$ when it receives an impulse $\lambda(-\mathbf{i}+\mathbf{j})\,\mathrm{N\,s}$, where λ is a positive constant.

$\lambda\begin{pmatrix}-1\\1\end{pmatrix}$

The angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse is $\theta°$

Immediately after receiving the impulse, P is moving with speed $4\sqrt{10}\,\mathrm{m\,s^{-1}}$

Find (i) the value of λ

(ii) the value of θ

(8)

(i) Impulse is the change in momentum M1

Formula for change in momentum:

$I = \Delta\text{momentum} = mv_{\text{final}} - mv_{\text{initial}}$ (mass; velocity)

$\therefore$ Formula for Impulse: $I = m(v-u)$

Let the velocity after be $\underset{\sim}{v} = \begin{pmatrix}p\\q\end{pmatrix}\mathrm{ms^{-1}}$

$I = 0.5\left(v - \begin{pmatrix}8\\0\end{pmatrix}\right)$

Equate the two impulses:

$0.5\left(v - \begin{pmatrix}8\\0\end{pmatrix}\right) = \lambda\begin{pmatrix}-1\\1\end{pmatrix}$

$0.5v - \begin{pmatrix}4\\0\end{pmatrix} = \lambda\begin{pmatrix}-1\\1\end{pmatrix}$

$0.5v = \lambda\begin{pmatrix}-1\\1\end{pmatrix} + \begin{pmatrix}4\\0\end{pmatrix}$

$0.5v = \begin{pmatrix}-\lambda+4\\\lambda\end{pmatrix}$

A1 $v = \begin{pmatrix}-2\lambda+8\\2\lambda\end{pmatrix}$ speed after

Use Pythagoras' theorem to get magnitude + equate to given speed.

given. — $4\sqrt{10} = \sqrt{(-2\lambda+8)^2+(2\lambda)^2}$ M1A1

$(4\sqrt{10})^2 = (-2\lambda+8)^2 + (2\lambda)^2$

Solve for λ: $160 = 4\lambda^2 - 32\lambda + 64 + 4\lambda^2$

$160 = 8\lambda^2 - 32\lambda + 64$

$0 = 8\lambda^2 - 32\lambda - 96$ $\div 8$

$0 = \lambda^2 - 4\lambda - 12$ factorize

$0 = (\lambda-6)(\lambda+2)$ M1

A1 $\lambda = 6$ ~~$\lambda = -2$~~ Reject negative

value of λ

(ii) Use scalar product formula:

$\cos\theta = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$

$\cos\theta = \dfrac{\begin{pmatrix}8\\0\end{pmatrix}\cdot\begin{pmatrix}-2(6)+8\\2(6)\end{pmatrix}}{8\times\sqrt{(-4)^2+12^2}}$ -4, 12 M1

$\theta = \cos^{-1}\left(\dfrac{-32}{8\times\sqrt{160}}\right)$

$\theta = 108°$ to 3sf value of θ A1

Question 4 continued

(Total for Question 4 is 8 marks)

5.

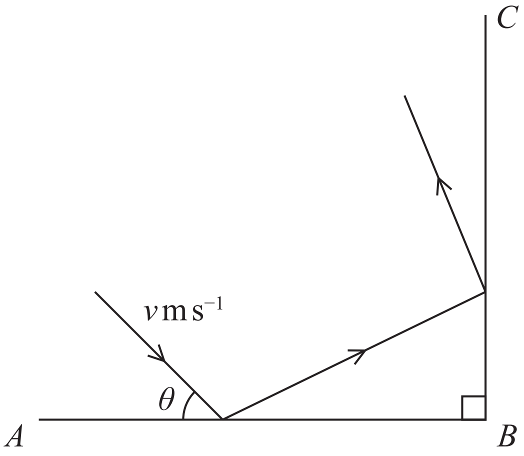


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls, with AB perpendicular to BC.

A small ball is projected along the floor towards the wall AB. Immediately before hitting the wall AB the ball is moving with speed $v\,\text{m s}^{-1}$ at an angle θ to AB.

The ball hits the wall AB and then hits the wall BC.

The coefficient of restitution between the ball and the wall AB is $\frac{1}{3}$

The coefficient of restitution between the ball and the wall BC is e.

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall AB.

(a) Find the exact value of $\cos\theta$.

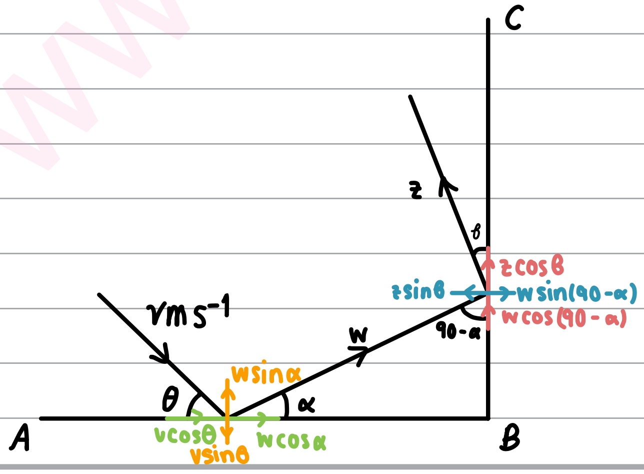
(5)

The ball loses half of its remaining kinetic energy in the impact with the wall BC.

(b) Find the exact value of e.

(5)

Diagram



· Parallel to the walls the components stay the same.

AB: $v\cos\theta = w\cos\alpha$

cos(90 - α) = sinα

BC: $w\cos(90-\alpha) = z\cos\beta$

$w\sin\alpha = z\cos\beta \rightarrow z\cos\beta = -\frac{1}{3}v\sin\theta$

for (b)

· Perpendicular to the wall use Impact law:

AB: $\frac{1}{3}v\sin\theta = -w\sin\alpha$

sin(90 - α) = cosα

BC: $e\,w\sin(90-\alpha) = -z\sin\beta$

$e\,w\cos\alpha = -z\sin\beta$

$e\,v\cos\theta = -z\sin\beta$ (use parallel from collision AB to equate)

for (b)

P 6 6 8 0 0 A 0 1 6 2 8

Question 5 continued

(a) We're given that $KE_F = \frac{1}{2} KE_I$

Formula for Kinetic Energy:

$$KE = \frac{1}{2} m v^2$$

mass, velocity

consider the first collision only:

$$\frac{1}{2} \times \frac{1}{2} m v^2 = \frac{1}{2} m w^2$$

We need to get w^2. Use the components we calculated on the previous page.

$w\sin\alpha = -\frac{1}{3} v\sin\theta$ B1B1

$w\cos\alpha = v\cos\theta$ see above how we got this

We can apply Pythagoras' Theorem to get w^2 in terms of $\sin\theta$ and $\cos\theta$:

$$w = \sqrt{(v\cos\theta)^2 + (-\tfrac{1}{3} v\sin\theta)^2}$$

$$w^2 = v^2\cos^2\theta + \frac{1}{9} v^2 \sin^2\theta$$

Substitute back into KE and solve for $\cos\theta$:

$$\frac{1}{2} \times \frac{1}{2} m v^2 = \frac{1}{2} m \left(v^2\cos^2\theta + \frac{1}{9} v^2 \sin^2\theta\right) \quad \text{M1}$$

$$\frac{1}{4} \cancel{mv^2} = \frac{1}{2} \cancel{mv^2} \cos^2\theta + \frac{1}{18} \cancel{mv^2} \sin^2\theta \quad \text{cancel } mv^2$$

$$\frac{1}{4} = \frac{1}{2}\cos^2\theta + \frac{1}{18}\sin^2\theta \quad \text{use identity } \sin^2\theta + \cos^2\theta = 1$$

$$\frac{1}{4} = \frac{1}{2}\cos^2\theta + \frac{1}{18}(1 - \cos^2\theta)$$

$$\frac{1}{4} = \frac{1}{2}\cos^2\theta + \frac{1}{18} - \frac{1}{18}\cos^2\theta \quad \times 2 \text{ both sides}$$

$$\frac{1}{2} = \cos^2\theta + \frac{1}{9} - \frac{1}{9}\cos^2\theta$$

$$\text{M1} \quad \frac{9}{18} - \frac{2}{18} = \frac{8}{9}\cos^2\theta \quad \text{collect like terms}$$

$$\frac{7}{18} = \frac{16}{18}\cos^2\theta$$

$$\frac{7}{16} = \cos^2\theta$$

$$\text{A1} \quad \cos\theta = \frac{\sqrt{7}}{4} \quad \text{value of } \cos\theta$$

Question 5 continued

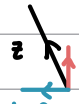
(b) The method is basically the same as in (a)

consider the second collision

$$KE_F = \frac{1}{2}KE_I$$

$$\frac{1}{2}\times\frac{1}{2}mw^2 = \frac{1}{2}mz^2$$

We need to get z^2. Use the components we calculated above:



B1B1

$$z\cos\theta = w\sin\alpha = -\frac{1}{3}v\sin\theta$$

$$z\sin\theta = -ew\cos\alpha = ev\cos\theta$$

We can apply Pythagoras' Theorem to get z^2 in terms of $\sin\theta$ and $\cos\theta$:

$$z = \sqrt{(ev\cos\theta)^2 + (-\tfrac{1}{3}v\sin\theta)^2}$$

$$z^2 = e^2v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta$$

Substitute back into KE and solve for e:

M1 $$\frac{1}{4}\cancel{m}(\underbrace{v^2\cos^2\theta + \tfrac{1}{9}v^2\sin^2\theta}_{w^2}) = \frac{1}{2}\cancel{m}(e^2v^2\cos^2\theta + \tfrac{1}{9}v^2\sin^2\theta)$$ cancel m's

$$\frac{1}{4}\cancel{v^2}(\cos^2\theta + \tfrac{1}{9}\sin^2\theta) = \frac{1}{2}\cancel{v^2}(e^2\cos^2\theta + \tfrac{1}{9}\sin^2\theta)$$ cancel v^2

$$\cos^2\theta + \frac{1}{9}\sin^2\theta = 2e^2\cos^2\theta + \frac{2}{9}\sin^2\theta$$ use $\sin^2 = 1-\cos^2\theta$ trig. Identity

$$\cos^2\theta + \frac{1}{9}(1-\cos^2\theta) = 2e^2\cos^2\theta + \frac{2}{9}(1-\cos^2\theta)$$ Substitute $\cos^2\theta = \frac{7}{16}$ from (a)

$$\frac{7}{16} + \frac{1}{9}(1-\tfrac{7}{16}) = 2e^2 \times \frac{7}{16} + \frac{2}{9}(1-\tfrac{7}{16})$$ solve for e

$$\frac{7}{16} + \frac{1}{9}\left(\frac{9}{16}\right) = \frac{7}{8}e^2 + \frac{2}{9}\left(\frac{9}{16}\right)$$

$$\frac{7}{16} + \frac{1}{16} = \frac{7}{8}e^2 + \frac{1}{8}$$ M1

$$\frac{8}{16} = \frac{7}{8}e^2 + \frac{1}{8}$$

$$\frac{1}{2} - \frac{1}{8} = \frac{7}{8}e^2$$

$$\frac{3}{8}\times\frac{8}{7} = e^2$$

$$e^2 = \frac{3}{7}$$

A1 $e = \sqrt{\frac{3}{7}}$ exact value of e

Question 5 continued

(Total for Question 5 is 10 marks)

6.

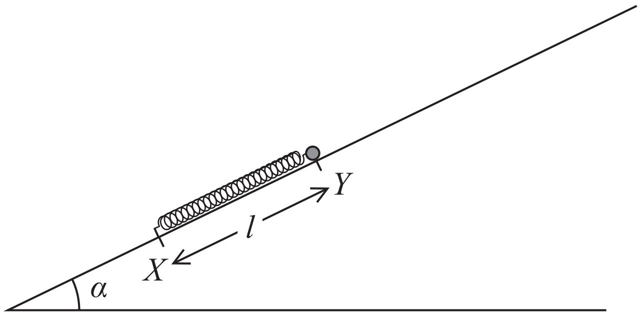


Figure 2

A light elastic spring has natural length $3l$ and modulus of elasticity $3mg$.

One end of the spring is attached to a fixed point X on a rough inclined plane.

The other end of the spring is attached to a package P of mass m.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The package is initially held at the point Y on the plane, where $XY = l$. The point Y is higher than X and XY is a line of greatest slope of the plane, as shown in Figure 2.

The package is released from rest at Y and moves up the plane.

The coefficient of friction between P and the plane is $\frac{1}{3}$ μ

By modelling P as a particle,

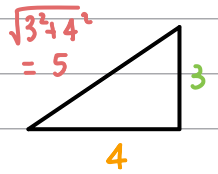
(a) show that the acceleration of P at the instant when P is released from rest is $\frac{17}{15}g$

(5)

(b) find, in terms of g and l, the speed of P at the instant when the spring first reaches its natural length of $3l$.

(6)

$\tan\alpha = \frac{3}{4}$

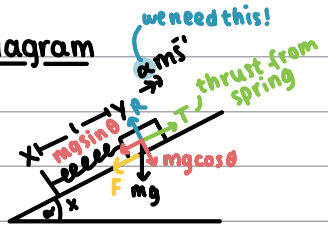


$\therefore \cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

Question 6 continued

(a) Diagram



Since we're looking for acceleration, use $\Sigma F_x = ma$

$T - F - mg\sin\theta = ma$ M1

We need to find:

T: formula for tension/thrust: compression. Natural length 3l, but XY = l.

$T = \frac{\lambda x}{l}$ $3l - l = 2l$ ← compression

$T = \frac{\cancel{3}mg(2\cancel{l})}{\cancel{3l}} = 2mg$ B1

F: as it's moving, F is maximum and $F = \mu R$

$F = \frac{1}{3} \times mg\cos\theta$ ← get this by doing $\Sigma F_y = 0$ vertically

$F = \frac{1}{3} mg\cos\theta$

Substitute back:

$2\cancel{m}g - \frac{1}{3}\cancel{m}g \times \frac{4}{5} - \cancel{m}g \times \frac{3}{5} = \cancel{m}a$ cancel m's A1

$2g - \frac{4}{15}g - \frac{3}{5}g = a$ A1

$a = \frac{17g}{15}$ value of a A1

Question 6 continued

(b) We will use the work-energy principle.

★ Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body (e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body (e.g. friction).

★ Remember the work-energy formulae:

Either: WD by force + KE_i + GPE_i + EPE_i = KE_F + GPE_F + EPE_F + WD against friction

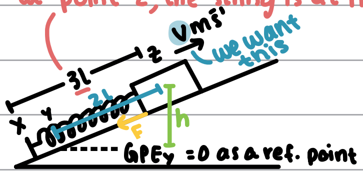
work done, initial kinetic, initial grav. potential, initial elastic potential, final kinetic, final grav. potential, final elastic potential, work lost to friction

OR: WD by force + KE_i + GPE_i + EPE_i − WD by friction = KE_F + GPE_F + EPE_F

work done, initial kinetic, initial grav. potential, initial elastic potential, we subtract this since it leaves the system as heat!, final kinetic, final grav. potential, final elastic potential

Diagram

at point z, the string is at natural length ∴ EPE = 0



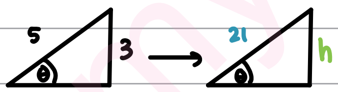
→ We are looking at motion from y to z.

Substitute into W-E principle:

$$\underbrace{\cancel{\tfrac{1}{2}m(0)^2} + \cancel{mg(0)} + \frac{3mg(2l)^2}{2\times 3l}}_{\text{at } y} - \underset{\text{distance moved}}{2l} \times F = \underbrace{\tfrac{1}{2}mv^2 + mgh + \cancel{\frac{3mg(0)}{2\times 3l}}}_{\text{at } z} \quad \text{M1}$$

We need h in terms of l.

given that:



$\sin\theta = \frac{3}{5}$ — $\sin\theta = \frac{h}{2l}$ → $\frac{3}{5} = \frac{h}{2l}$ $h = \frac{6}{5}l$

Substitute back:

$$\frac{3mg\times 4l^2}{6l} - 2l\times\left(\tfrac{1}{3}mg\cos\theta\right) = \tfrac{1}{2}mv^2 + mg\left(\tfrac{6}{5}l\right) \quad \text{B1B1B1}$$

$$\frac{\overset{2}{\cancel{12}}mgl^{\cancel{2}}}{\cancel{6l}} - \tfrac{2}{3}l\times\tfrac{4}{5}mg = \tfrac{1}{2}mv^2 + \tfrac{6}{5}mgl \quad \text{A1}$$

$$2\cancel{m}gl - \tfrac{8}{15}\cancel{m}gl - \tfrac{6}{5}\cancel{m}gl = \tfrac{1}{2}\cancel{m}v^2 \quad \text{cancel m's}$$

$$\tfrac{30}{15}gl - \tfrac{8}{15}gl - \tfrac{18}{15}gl = \tfrac{1}{2}v^2$$

$$\tfrac{4}{15}gl = \tfrac{1}{2}v^2$$

$$\tfrac{8}{15}gl = v^2$$

$$v = \sqrt{\frac{8gl}{15}} \quad \text{value of } V \quad \text{A1}$$

P 6 6 8 0 0 A 0 2 2 2 8

Question 6 continued

(Total for Question 6 is 11 marks)

7. [*In this question*, **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

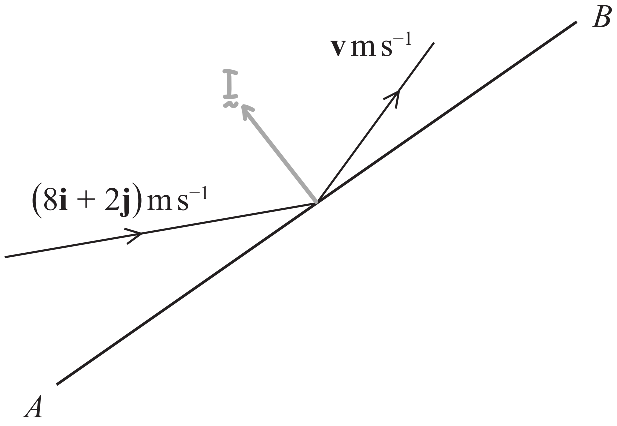


Figure 3

Figure 3 represents the plan view of part of a smooth horizontal floor, where AB is a fixed smooth vertical wall.

The direction of $\overrightarrow{AB}$ is in the direction of the vector $(\mathbf{i} + \mathbf{j})$

A small ball of mass 0.25 kg is moving on the floor when it strikes the wall AB.

Immediately before its impact with the wall AB, the velocity of the ball is $(8\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$

Immediately after its impact with the wall AB, the velocity of the ball is $\mathbf{v}\,\mathrm{m\,s^{-1}}$

The coefficient of restitution between the ball and the wall is $\frac{1}{3}$ e

By modelling the ball as a particle,

(a) show that $\mathbf{v} = 4\mathbf{i} + 6\mathbf{j}$

(6)

(b) Find the magnitude of the impulse received by the ball in the impact.

(3)

Question 7 continued

(a) Formulae for vector collisions:

$\underset{\sim}{u}\cdot\underset{\sim}{w} = \underset{\sim}{v}\cdot\underset{\sim}{w}$ → parallel vector to wall

initial speed   final speed

$-e\,\underset{\sim}{u}\cdot\underset{\sim}{I} = \underset{\sim}{v}\cdot\underset{\sim}{I}$ → perpendicular vector to wall

Let's write down what we know:

$\underset{\sim}{u} = \begin{pmatrix}8\\2\end{pmatrix}\text{ms}^{-1}$ $\underset{\sim}{w} = \begin{pmatrix}1\\1\end{pmatrix}$ — in this case it's easy to

$\underset{\sim}{v} = \begin{pmatrix}p\\q\end{pmatrix}\text{ms}^{-1}$ $\underset{\sim}{I} = \begin{pmatrix}-1\\1\end{pmatrix}$ ← get the perpendicular

$e = \frac{1}{3}$ vector, just make i or j negative.

Substitute into the formulae:

parallel $\begin{pmatrix}8\\2\end{pmatrix}\cdot\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}p\\q\end{pmatrix}\cdot\begin{pmatrix}1\\1\end{pmatrix}$ M1

$8+2 = p+q$

$10 = p+q$ Eq1 A1

perpendicular $-\frac{1}{3}\times\begin{pmatrix}8\\2\end{pmatrix}\cdot\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}p\\q\end{pmatrix}\cdot\begin{pmatrix}-1\\1\end{pmatrix}$ M1

$-\frac{1}{3}\times(-8+2) = -p+q$

$-\frac{1}{3}\times -6 = q-p$

$2 = q-p$ Eq.2 A1

Solve simultaneously Eq1 and Eq2:

$10 = p+q$

$2 = -p+q$ +

$12 = 2q$

$q = 6,\ p = 4$

$\therefore\ \underset{\sim}{v} = \begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}4\\6\end{pmatrix} = (4\mathbf{i}-6\mathbf{j})\text{ms}^{-1}$ shown M1A1

(b) Impulse is the change in momentum

Formula for change in momentum:

$I = \Delta\text{momentum} = mv_{\text{final}} - mv_{\text{initial}}$ mass   velocity

$\therefore$ Formula for Impulse: $I = m(v-u)$

Substitute: $\underset{\sim}{I} = 0.25\left(\begin{pmatrix}4\\6\end{pmatrix} - \begin{pmatrix}8\\2\end{pmatrix}\right)$

$= 0.25\times\begin{pmatrix}-4\\4\end{pmatrix}$

$\underset{\sim}{I} = \begin{pmatrix}-1\\1\end{pmatrix}$ M1

To get the magnitude use Pythagoras' theorem:

$|\underset{\sim}{I}| = \sqrt{(-1)^2+(1)^2}$

$= \sqrt{1+1}$

$= \sqrt{2}$ Ns magnitude of Impulse M1A1

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS